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MODEL OF TURBULENT OSCILLATING FLOWS IN SMOOTH TUBES

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Turbulent oscillating flows in smooth tubes are considered. A group of flows in which the logarithmic boundary layer grows monotonically with time is distinguished; the conditions of quasistationarity of these flows are determined. A model of a quasistationary turbulent oscillating flow in a smooth tube is constructed; the model is in satisfactory agreement with experiment.

Turbulent oscillating flows in smooth tubes were the subject of a number of studies [1–7]. Among the works considered, of greatest interest are [3, 6], in which flows with a logarithmic boundary layer, whose thickness δ grows monotonically with time until the layer occupies the entire cross section of the tube, are studied. The velocity maximum, until the logarithmic layer reaches the channel axis, is observed at a distance $y = \delta$, and it is markedly higher than on the tube axis where the velocity changes according to the law $U_0 = U_{0m} \sin \omega t$.

Integral relations for nonstationary boundary layers can be derived from the boundary-layer equations using the same technique as for stationary ones. The solution of them is given in [2]. Another technique [7] is approximate integration of the boundary-layer equations. In this case, one has to introduce the assumption of the constancy of the coefficient of eddy viscosity with time, which obviously does not correspond to the experiment.

In the work, an attempt is made to determine conditions under which the mentioned flow is realized and to suggest the corresponding model.

In harmonic oscillations of a plate in its Stokes plane, the Stokes wave propagates into the depth of liquid [8] with a velocity $v = dy/dt$ determined by the formula

$$\frac{dy}{dt} = \sqrt{2\nu\omega}\ . \tag{1}$$

In the case of a turbulized medium we can expect

$$\frac{dy}{dt} = \sqrt{2\,(\nu + \nu_t)\,\omega}\ . \tag{2}$$

Usually $\nu_t >> \nu$; therefore the velocity of the wave will be determined mainly by the eddy viscosity. In the near-wall region, $\nu_t = \kappa y v^*$ [8].

We assume that the tangential stress on the wall changes according to the harmonic law $\tau_w = \tau_{w0} \sin (\omega t + \phi)$, and after integration of (2) we obtain

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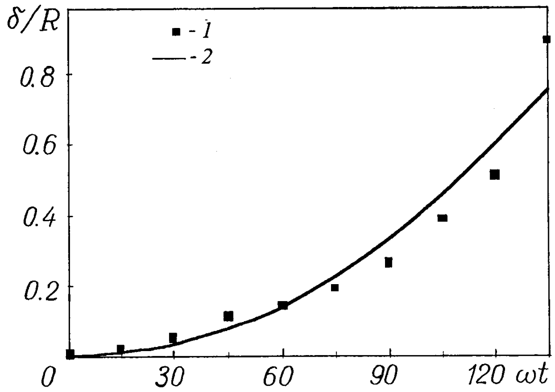

Fig. 1. Dependence of the ratio of the boundary-layer thickness to the tube radius on the phase of oscillation: 1) experimental data of [6]; 2) theory according to [6]. ωt, deg.

$$\delta(t) = \frac{\kappa v_0^*}{2\omega} f(\varphi) , \quad f(\varphi) = \left(\int_0^{\varphi} \sin^{1/4} \tau d\tau \right)^2 , \quad \varphi = \omega t + \phi . \tag{3}$$

Since near the wall the behavior of flows is the same for both the plane and three-dimensional cases, Eqs. (1)–(3) can be used for describing a tube flow.

According to [6], for large Reynolds numbers $\phi \approx 0.1308$. For the flow to be quasistationary, the equality $\delta/R \approx 1$ must be reached at earlier stages of acceleration.

Figure 1 presents the dependences of δ/R on the phase of oscillation for the parameters of the setup [6]. The points indicate the corresponding experimental data [6]. It is seen that relation (3) is in qualitative agreement with the experiment [6].

As is known, turbulent flows are dissipative; therefore, a constant supply of energy is needed to maintain turbulence. At the same time, owing to the turbulent motion, diffusion of particles and their kinetic energy occur. The steady state is observed in the case of equilibrium between the supplied energy (turbulence generation) and the diffusion and dissipation of turbulence energy. In the unsteady state, this equilibrium is not observed and any excess of energy must be equal to a change in the turbulence energy [9].

If the flow is bounded by solid walls (the flow velocity on the walls vanishes), then in integration over the entire region of flow all terms expressing turbulent and molecular diffusion disappear. We restrict ourselves to plane thin-layer flows. Then we can write

$$\frac{D\left[\left(\frac{1}{2}\rho q^2\right)\right]}{Dt} = \left[-\rho \overline{u'v'} \frac{\partial U}{\partial y} \right] - [\varepsilon_t] , \tag{4}$$

where $q^2 = \overline{u'^2} + \overline{v'^2} + \overline{w'^2}$. The left-hand side of (4) has the rate of change of the kinetic energy. The first term on the right-hand side is the rate of generation of the turbulence energy, and the second is the turbulent part of dissipation. Here it is taken that the molecular dissipation is much smaller than the turbulent dissipation. In an oscillating flow we can take $D/Dt = \partial/\partial t$. We introduce the notation

$$E = \frac{1}{2} \rho \int_0^R \left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right) dy , \quad P = \left[-\rho \overline{u'v'} \frac{\partial U}{\partial y} \right] = -\rho \int_0^R \overline{u'v'} \frac{\partial U}{\partial y} dy ; \tag{5}$$

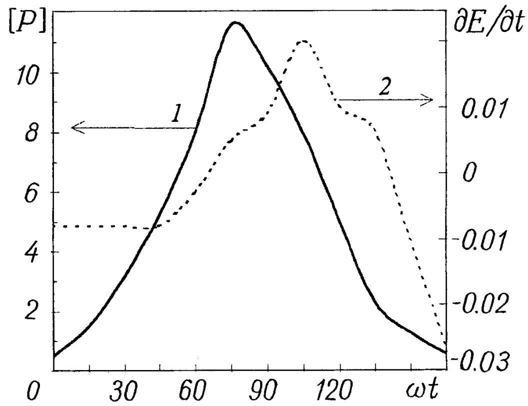

Fig. 2. Dependence of the turbulence-energy generation (1) and the change in the kinetic energy of turbulence (2) on the time from the data of [6]. P, $\partial E/\partial t$, J/sec.

then (4) takes the form

$$\frac{\partial E}{\partial t} = P - [\varepsilon_t] . \tag{6}$$

The values of P and $[\varepsilon_t]$ in stationary flows must be equal; therefore the ratio $(\partial E/\partial t)/P$ is the measure of nonstationarity of the considered flow.

Figure 2 presents the results of numerical integration of the data of [6] by expression (5). The same figure gives the values of $\partial E/\partial t$. It is seen that the maximum of P falls at the phase of 75^o. The kinetic energy of turbulence E at the stage of acceleration has a tendency to decrease (negative values of $\partial E/\partial t$), and beginning from $\omega t = 90^o$ to increase. Except for extreme points ($\omega t \approx 0^o$ and $\omega t \approx 165^o$), the value of $\partial E/\partial t$ is much smaller than P. This indicates that change in the kinetic energy of turbulence in oscillating flows does not contribute markedly to the energy balance.

We consider the energy equation [9]

$$\varepsilon_t = -\overline{u'v'}\frac{dU}{dy} - \frac{d}{dy}\left[\nu\left(\frac{p}{\rho} + \frac{1}{2}q^2\right)\right] . \tag{7}$$

To reduce it to an integrable form, we must express all terms of this equation in terms of the tangential stress τ:

$$-\overline{u'v'} = \frac{\tau}{\rho} , \quad \nu\left(\frac{p}{\rho} + \frac{1}{2}q^2\right) = B_1\left(\frac{\tau}{\rho}\right)^{3/2} , \quad \varepsilon_t = \frac{A}{y}\left(\frac{\tau}{\rho}\right)^{1/2} . \tag{8}$$

In stationary flows, we have $3B_1 \approx -2A$ and $A = 1/\kappa$. Then Eq. (7) takes the form

$$\frac{dU}{dy} = \frac{A}{y}\left(\frac{\tau}{\rho}\right)^{1/2}\left(1 + \frac{3B_1}{2A}\frac{y}{\tau}\frac{d\tau}{dy}\right) . \tag{9}$$

Equation (9) has the advantage that the eddy viscosity does not appear in it explicitly.

To construct the model corresponding to the experiments of [6] (curve in Fig. 1), it is worthwhile to divide the flow into two regions: the near-wall region ($0 < y < \delta$) and the core ($\delta < y < R$).

We consider the near-wall region, which, in turn, can be divided into two stages: acceleration and retardation. In the stage of acceleration, the maximum of velocity U_m is observed at a distance $y = \delta$ from the wall rather than on the tube axis; in this case $U_m > U_0$. Since at the point of velocity maximum the tangential stress vanishes, there is good reason to believe that, similarly to stationary flows [9],

$$\frac{d\tau}{dy} = \frac{dp}{dx}, \tag{10}$$

whence

$$\tau = \tau_w + \frac{dp}{dx} y, \tag{11}$$

where $dp/dx = -\rho\omega U_{0m} \cos \omega t$.

At the distance δ from the wall the tangential stress τ must vanish. However, calculation of δ in accordance with (11) gives substantially underestimated values, as compared to experiment although the linear character of change of $\tau = \tau(y)$ remains. Therefore, in the stage of acceleration, instead of dp/dx in (11) we should take the parameter α, calculated proceeding from the condition $\tau = 0$, must be taken for $y = \delta$:

$$\tau = \tau_w - \alpha y, \quad \alpha = \tau_w/\delta. \tag{12}$$

We substitute (12) into (9) and integrate

$$\frac{U}{v^*} = (2A + 3B_1)(1 - y/\delta)^{1/2} + A \ln\left(\frac{1 - \sqrt{1 - y/\delta}}{1 + \sqrt{1 - y/\delta}}\right) + C. \tag{13}$$

Since for $y/\delta = 1$ the first and second terms on the right-hand side vanish, we must have $U_m/v^* = C$. To determine C, we take into account the fact that for small y/δ expression (13) must coincide with the logarithmic law obtained for an equilibrium turbulent boundary layer. Since at the point $y_1 = \nu/v^*$, $\ln(v^* y_1/\nu)$ vanishes, we have

$$C = A_{\text{theor}} \ln\left(\frac{\delta v^*}{\nu}\right) + B - (2 - \ln 4) A_{\text{theor}} - 3B_1. \tag{14}$$

In the retardation stage, including $\omega t = 90^\circ$, the tangential stress has a more complex dependence on $\xi = y/\delta$ than (11). We express it in the form of a polynomial which allows for the effect of the positive gradient of pressure. The necessary conditions have the form [10]

$$\tau = \tau_w, \ \ \xi = 0; \ \ \tau \sim \tau_w (1 + \Lambda\xi), \ \ \xi \to 0; \ \ \tau = 0, \ \ \partial\tau/\partial\xi = 0, \ \ \xi = 1, \tag{15}$$

where $\Lambda = (\delta/\tau_w)(dp/dx)$.

The cubic parabola

$$\tau = \tau_w (1 - 3\xi^2 + 2\xi^3 + \Lambda\xi(1 - \xi)^2), \quad \Lambda = -\frac{\rho\delta\omega U_{0m} \cos \omega t}{\tau_w}. \tag{16}$$

satisfies conditions (15). Substitution of (16) into (9) and integration allow one to obtain the following expression for the velocity in the retardation stage:

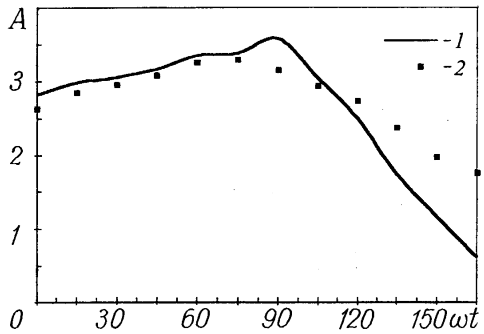

Fig. 3. Coefficient A vs. time: 1) theory; 2) experiment [6].

$$\frac{U}{v^*} = A_{\text{theor}} \left(\int_{y_1/\delta}^{1} \frac{\sqrt{(1 - 3\xi^2 + 2\xi^3) + \Lambda_1 \xi (1 - \xi^2)}}{\xi} d\xi \right) + 3B_1 + B \,, \tag{17}$$

where $y_1 = \nu/v^*$.

To compare the results of (13), (14), and (17) we proceed as follows. We calculate experimental values of A_{exp} from the data of [6] (this is admitted since experimental profiles obey with good accuracy, the logarithmic law; where $A = A_{\text{exp}}$, which differs from the universal law, here $A = 2.5$, but with a constant value $B = 1.5$). Assuming that the maximum theoretical and experimental values of the velocity U_{m} must coincide, we calculate A_{theor}. Results of the calculation of A_{theor} are shown in Fig. 3 by a solid line; points correspond to A_{exp}. Good qualitative agreement of the results is seen. Quantitative divergences for $\omega t < 135^{\circ}$ lie within 20%. With increase in the phase, in the subsequent phase of retardation ($\omega t > 135^{\circ}$), the divergences between theory and experiment begin to increase, which is caused by the inadequacy of the boundary condition $\tau \sim \tau_{\text{w}}(1 + \Lambda\xi)$ to the actual behavior of the tangential stress near the wall [11].

We find the velocity in the flow core, for which purpose we again use its representation in the form of a polynomial. The set of conditions has the form

$$U = U_{\text{m}} \,, \quad \tau = 0 \,, \quad y = \delta \,; \quad U = U_0 \,, \quad \frac{\partial \tau}{\partial y} = 0 \,, \quad \tau = 0 \,, \quad y = R \,. \tag{18}$$

They correspond to the polynomial

$$U - U_0 = (U_{\text{m}} - U_0) \, (1 - 6\eta^2 + 8\eta^3 - 3\eta^4) \,, \ \ \eta = \frac{y - \delta}{R - \delta} \,. \tag{19}$$

As is seen, the velocity distribution in the flow core coincides with the distribution in free jets [12].

NOTATION

x, longitudinal coordinate; y, distance reckoned from the wall to the normal; ξ and η, dimensionless coordinates; δ, thickness of the viscous boundary layer; R, tube radius; U, velocity of the averaged flow; $U_{0\text{m}}$, amplitude of velocity oscillations on the tube axis; U_0, velocity on the tube axis; U_{m}, maximum velocity; v, transverse component of the velocity; u', v', and w', pulsating components of the velocity; $v^* = \sqrt{\tau_{\text{w}}/\rho}$, rate of tangential stress; $v_0^* = \sqrt{\tau_{\text{w0}}/\rho}$, amplitude of the rate of tangential stress; ω, cyclic frequency of oscillations; t, time; ϕ, initial phase; ν, kinematic viscosity; ν_{t}, eddy viscosity; τ, tangential stress; τ_{w}, tangential stress on the wall; τ_{w0}, amplitude of tangential stress on the wall; p, pressure; ρ, density of the liquid; ρ_0, density of the gas; E, kinetic energy of turbulence; P, generation of turbulence energy; $[\varepsilon_{\text{t}}]$, turbulent part of dissipation, brackets denote averaging over the channel cross section; $A = 1/\kappa$, parameter obeying the universal law, $\kappa = 0.4$ is the von Kármán constant; A_{exp}, experimental value of the parameter obeying the logarithmic law; A_{theor}, theoretical value of the parameter for the logarithmic law; $B = 1.5$, constant determined from the profile of comparison; B_1, constant.

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